CHAPTER 1

NUMBER SYSTEMS

(A) Main Concepts and Results

Rational numbers

Irrational numbers

Locating irrational numbers on the number line

Real numbers and their decimal expansions

Representing real numbers on the number line

Operations on real numbers

Rationalisation of denominator

Laws of exponents for real numbers

- A number is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- A number which cannot be expressed in the form $\frac{p}{q}$ (where p and q are integers and $q \neq 0$) is called an irrational number.
- All rational numbers and all irrational numbers together make the collection of real numbers.
- Decimal expansion of a rational number is either terminating or non-terminating recurring, while the decimal expansion of an irrational number is non-terminating non-recurring.

- If r is a rational number and s is an irrational number, then $r+s$ and $r-s$ are irrationals.

  Further, if r is a non-zero rational, then rs and $\frac{r}{s}$ are irrationals.

- For positive real numbers a and b :

  (i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

  (iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$ (iv) $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

  (v) $(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$

- If p and q are rational numbers and a is a positive real number, then

  (i) $a^p \,.\, a^q = a^{p+q}$ (ii) $(a^p)^q = a^{pq}$

  (iii) $\frac{a^p}{a^q} = a^{p-q}$ (iv) $a^p b^p = (ab)^p$

(B) Multiple Choice Questions

Write the correct answer:

Sample Question 1 : Which of the following is not equal to $\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}}$?

(A) $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$ (B) $\dfrac{1}{\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{\frac{1}{6}}}$ (C) $\left(\frac{6}{5}\right)^{\frac{1}{30}}$ (D) $\left(\frac{5}{6}\right)^{-\frac{1}{30}}$

Solution : Answer (A)

EXERCISE 1.1

Write the correct answer in each of the following:

1. Every rational number is

(A) a natural number (B) an integer

(C) a real number (D) a whole number

2. Between two rational numbers
- (A) there is no rational number
- (B) there is exactly one rational number
- (C) there are infinitely many rational numbers
- (D) there are only rational numbers and no irrational numbers

3. Decimal representation of a rational number cannot be
- (A) terminating
- (B) non-terminating
- (C) non-terminating repeating
- (D) non-terminating non-repeating

4. The product of any two irrational numbers is
- (A) always an irrational number
- (B) always a rational number
- (C) always an integer
- (D) sometimes rational, sometimes irrational

5. The decimal expansion of the number $\sqrt{2}$ is
- (A) a finite decimal
- (B) 1.41421
- (C) non-terminating recurring
- (D) non-terminating non-recurring

6. Which of the following is irrational?

(A) $\sqrt{\frac{4}{9}}$ (B) $\frac{\sqrt{12}}{\sqrt{3}}$ (C) $\sqrt{7}$ (D) $\sqrt{81}$

7. Which of the following is irrational?

(A) 0.14 (B) $0.14\overline{16}$ (C) $0.\overline{1416}$ (D) 0.4014001400014...

8. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$ (C) 1.5 (D) 1.8

9. The value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(A) $\frac{19}{10}$ (B) $\frac{1999}{1000}$ (C) 2 (D) $\frac{1}{9}$

10. $2\sqrt{3}+\sqrt{3}$ is equal to

(A) $2\sqrt{6}$ (B) 6 (C) $3\sqrt{3}$ (D) $4\sqrt{6}$

11. $\sqrt{10}\times\sqrt{15}$ is equal to

(A) $6\sqrt{5}$ (B) $5\sqrt{6}$ (C) $\sqrt{25}$ (D) $10\sqrt{5}$

12. The number obtained on rationalising the denominator of $\frac{1}{\sqrt{7}-2}$ is

(A) $\frac{\sqrt{7}+2}{3}$ (B) $\frac{\sqrt{7}-2}{3}$ (C) $\frac{\sqrt{7}+2}{5}$ (D) $\frac{\sqrt{7}+2}{45}$

13. $\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(A) $\frac{1}{2}\left(3-2\sqrt{2}\right)$ (B) $\frac{1}{3+2\sqrt{2}}$

(C) $3-2\sqrt{2}$ (D) $3+2\sqrt{2}$

14. After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(A) 13 (B) 19 (C) 5 (D) 35

15. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to

(A) $\sqrt{2}$ (B) 2 (C) 4 (D) 8

16. If $\sqrt{2}$ = 1.4142, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(A) 2.4142 (B) 5.8282
(C) 0.4142 (D) 0.1718

17. $\sqrt[4]{\sqrt[3]{2^2}}$ equals

(A) $2^{-\frac{1}{6}}$ (B) 2^{-6} (C) $2^{\frac{1}{6}}$ (D) 2^6

18. The product $\sqrt[3]{2}\cdot\sqrt[4]{2}\cdot\sqrt[12]{32}$ equals

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt[12]{2}$ (D) $\sqrt[12]{32}$

19. Value of $\sqrt[4]{(81)^{-2}}$ is

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) 9 (D) $\frac{1}{81}$

20. Value of $(256)^{0.16}\times(256)^{0.09}$ is

(A) 4 (B) 16 (C) 64 (D) 256.25

21. Which of the following is equal to x?

(A) $x^{\frac{12}{7}}-x^{\frac{5}{7}}$ (B) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$ (C) $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$ (D) $x^{\frac{12}{7}}\times x^{\frac{7}{12}}$

(C) Short Answer Questions with Reasoning

Sample Question 1: Are there two irrational numbers whose sum and product both are rationals? Justify.

Solution : Yes.

$3+\sqrt{2}$ and $3-\sqrt{2}$ are two irrational numbers.

$\left(3+\sqrt{2}\right)+\left(3-\sqrt{2}\right)=6$, a rational number.

$\left(3+\sqrt{2}\right)\times\left(3-\sqrt{2}\right)=7$, a rational number.

So, we have two irrational numbers whose sum and product both are rationals.

Sample Question 2: State whether the following statement is true:

There is a number x such that x^2 is irrational but x^4 is rational. Justify your answer by an example.

Solution : True.

Let us take $x = \sqrt[4]{2}$

Now, $x^2 = \left(\sqrt[4]{2}\right)^2 = \sqrt{2}$, an irrational number.

$x^4 = \left(\sqrt[4]{2}\right)^4 = 2$, a rational number.

So, we have a number x such that x^2 is irrational but x^4 is rational.

EXERCISE 1.2

1. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number? Give an example in support of your answer.

2. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

3. State whether the following statements are true or false? Justify your answer.

(i) $\frac{\sqrt{2}}{3}$ is a rational number.

(ii) There are infinitely many integers between any two integers.

(iii) Number of rational numbers between 15 and 18 is finite.

(iv) There are numbers which cannot be written in the form $\frac{p}{q}$, $q \neq 0$, p, q both are integers.

(v) The square of an irrational number is always rational.

(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, $q \neq 0$ and so it is a rational number.

4. Classify the following numbers as rational or irrational with justification :

(i) $\sqrt{196}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{\frac{9}{27}}$ (iv) $\frac{\sqrt{28}}{\sqrt{343}}$

(v) $-\sqrt{0.4}$ (vi) $\dfrac{\sqrt{12}}{\sqrt{75}}$ (vii) 0.5918

(viii) $\left(1+\sqrt{5}\right)-\left(4+\sqrt{5}\right)$ (ix) 10.124124... (x) 1.010010001...

(D) Short Answer Questions

Sample Question 1: Locate $\sqrt{13}$ on the number line.

Solution : We write 13 as the sum of the squares of two natural numbers :

$13 = 9 + 4 = 3^2 + 2^2$

On the number line, take OA = 3 units.

Draw BA = 2 units, perpendicular to OA. Join OB (see Fig.1.1).

By Pythagoras theorem, OB = $\sqrt{13}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{13}$.

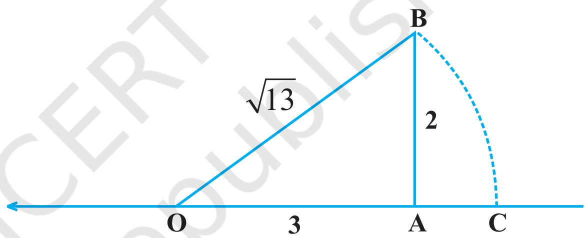

Fig. 1.1

Remark **:** We can also take OA = 2 units and AB = 3 units.

Sample Question 2 : Express $0.12\overline{3}$ in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

Solution :

Let $x = 0.12\overline{3}$

so, $10x = 1.2\overline{3}$

or $10x - x = 1.2\overline{3} - 0.12\overline{3} = 1.2333 \ldots - 0.12333 \ldots$

or $9x = 1.11$

or $x = \dfrac{1.11}{9} = \dfrac{111}{900}$

Therefore, $0.12\overline{3} = \frac{111}{900} = \frac{37}{300}$

Sample Question 3 : Simplify : $\left(3\sqrt{5}-5\sqrt{2}\right)\left(4\sqrt{5}+3\sqrt{2}\right)$.

Solution : $\left(3\sqrt{5}-5\sqrt{2}\right)\left(4\sqrt{5}+3\sqrt{2}\right)$

$$= 12\times5-20\sqrt{2}\times\sqrt{5}+9\sqrt{5}\times\sqrt{2}-15\times2$$

$$= 60-20\sqrt{10}+9\sqrt{10}-30$$

$$= 30-11\sqrt{10}$$

Sample Question 4 : Find the value of a in the following :

$$\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2}-a\sqrt{3}$$

Solution : $\frac{6}{3\sqrt{2}-2\sqrt{3}} = \frac{6}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$$= \frac{6\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2} = \frac{6\left(3\sqrt{2}+2\sqrt{3}\right)}{18-12} = \frac{6\left(3\sqrt{2}+2\sqrt{3}\right)}{6}$$

$$= 3\sqrt{2}+2\sqrt{3}$$

Therefore, $3\sqrt{2}+2\sqrt{3} = 3\sqrt{2}-a\sqrt{3}$

or $a = -2$

Sample Question 5: Simplify : $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

Solution :

$$\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$$

$$= \left[5(2+3)^3\right]^{\frac{1}{4}}$$

$$= \left[5\,(5)^3\right]^{\frac{1}{4}}$$

$$= [5^4]^{\frac{1}{4}} = 5$$

EXERCISE 1.3

1. Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = .04$ (iv) $u^2 = \frac{17}{4}$

2. Find three rational numbers between

(i) -1 and -2 (ii) 0.1 and 0.11

(iii) $\frac{5}{7}$ and $\frac{6}{7}$ (iv) $\frac{1}{4}$ and $\frac{1}{5}$

3. Insert a rational number and an irrational number between the following :

(i) 2 and 3 (ii) 0 and 0.1 (iii) $\frac{1}{3}$ and $\frac{1}{2}$

(iv) $\frac{-2}{5}$ and $\frac{1}{2}$ (v) 0.15 and 0.16 (vi) $\sqrt{2}$ and $\sqrt{3}$

(vii) 2.357 and 3.121 (viii) .0001 and .001 (ix) 3.623623 and 0.484848

(x) 6.375289 and 6.375738

4. Represent the following numbers on the number line :

7, 7.2, $\frac{-3}{2}$, $\frac{-12}{5}$

5. Locate $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{17}$ on the number line.

6. Represent geometrically the following numbers on the number line :

(i) $\sqrt{4.5}$ (ii) $\sqrt{5.6}$ (iii) $\sqrt{8.1}$ (iv) $\sqrt{2.3}$

7. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$:

(i) 0.2 (ii) 0.888... (iii) $5.\bar{2}$ (iv) $0.\overline{001}$

(v) 0.2555... (vi) $0.1\overline{34}$ (vii) .00323232... (viii) .404040...

8. Show that $0.142857142857... = \frac{1}{7}$

9. Simplify the following:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ (ii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12} \times \sqrt[7]{6}$ (iv) $4\sqrt{28} \div 3\sqrt{7} \div \sqrt[3]{7}$

(v) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$ (vi) $\left(\sqrt{3} - \sqrt{2}\right)^2$

(vii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$ (viii) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

(ix) $\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{6}$

10. Rationalise the denominator of the following:

(i) $\frac{2}{3\sqrt{3}}$ (ii) $\frac{\sqrt{40}}{\sqrt{3}}$ (iii) $\frac{3+\sqrt{2}}{4\sqrt{2}}$

(iv) $\frac{16}{\sqrt{41}-5}$ (v) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ (vi) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

(vii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ (viii) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

11. Find the values of a and b in each of the following:

(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - 6\sqrt{3}$

(ii) $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \dfrac{19}{11}$

(iii) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

(iv) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} - \dfrac{7-\sqrt{5}}{7+\sqrt{5}} = a + \dfrac{7}{11}\sqrt{5}b$

12. If $a = 2+\sqrt{3}$, then find the value of $a - \dfrac{1}{a}$.

13. Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

(i) $\dfrac{4}{\sqrt{3}}$ (ii) $\dfrac{6}{\sqrt{6}}$ (iii) $\dfrac{\sqrt{10}-\sqrt{5}}{2}$

(iv) $\dfrac{\sqrt{2}}{2+\sqrt{2}}$ (v) $\dfrac{1}{\sqrt{3}+\sqrt{2}}$

14. Simplify :

(i) $\left(1^3+2^3+3^3\right)^{\frac{1}{2}}$

(ii) $\dfrac{3}{5}^{4}\ \dfrac{8}{5}^{-12}\ \dfrac{32}{5}^{6}$

(iii) $\dfrac{1}{27}^{\frac{-2}{3}}$

(iv) $(625)^{-\frac{1}{2}}{}^{-\frac{1}{4}}{}^{2}$

(v) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

(vi) $64^{-\frac{1}{3}}\ 64^{\frac{1}{3}} - 64^{\frac{2}{3}}$

(vii) $\dfrac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

(E) Long Answer Questions

Sample Question 1 : If $a = 5 + 2\sqrt{6}$ and $b = \frac{1}{a}$, then what will be the value of $a^2 + b^2$?

Solution : $a = 5 + 2\sqrt{6}$

$$b = \frac{1}{a} = \frac{1}{5+2\sqrt{6}} = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{5-2\sqrt{6}}{5^2-(2\sqrt{6})^2} = \frac{5-2\sqrt{6}}{25-24} = 5-2\sqrt{6}$$

Therefore, $a^2 + b^2 = (a + b)^2 - 2ab$

Here, $a + b = (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) = 10$

$ab = (5 + 2\sqrt{6})(5 - 2\sqrt{6}) = 5^2 - (2\sqrt{6})^2 = 25 - 24 = 1$

Therefore, $a^2 + b^2 = 10^2 - 2 \times 1 = 100 - 2 = 98$

EXERCISE 1.4

1. Express $0.6 + 0.\bar{7} + 0.4\bar{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

2. Simplify : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

3. If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$.

4. If $a = \frac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$.

5. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$.

6. Simplify : $(256)^{-\left(4^{\frac{-3}{2}}\right)}$

7. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$