

UNIT 12

PRACTICAL GEOMETRY SYMMETRY AND VISUALISING SOLID SHAPES

(A) Main Concepts and Results

- Let a line ' l ' and a point P not lying on it be given. By using properties of a transversal and parallel lines, a line which passes through the point P and parallel to ' l ', can be drawn.
- A triangle can be drawn if any one of the following sets of measurements are given :
 - (i) Three sides (SSS).
 - (ii) Two sides and the angle between them (SAS).
 - (iii) Two angles and a side (AAS) or (ASA).
 - (iv) The hypotenuse and a leg in the case of a right-angled triangle (RHS).
- A figure has line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide with each other.
- Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry.
- Each regular polygon has as many lines of symmetry as it has sides.
- Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.

- Rotation turns an object about a fixed point. This fixed point is called the centre of rotation.
- The angle by which the object rotates is the angle of rotation. Rotation may be clockwise or anti-clockwise.
- A half-turn means rotation by 180° . A quarter-turn means rotation by 90° .
- If, after a rotation, a figure or an object coincides with the original position, we say that it has a rotational symmetry.
- In a complete turn (of 360°), the number of times the figure coincides with its original position is called its order of rotational symmetry.
- Every figure has a rotational symmetry of order 1 (i.e. a rotational symmetry of angle 360°). In such a case it is considered that the figure has no rotational symmetry.
- Some shapes have only one line of symmetry, like the letter E; some have only rotational symmetry, like the letter S; and some have both vertical and horizontal lines of symmetry, like the letter H.
- Plane figures are of two-dimensions (2-D) and the solid shapes are of three-dimensions (3-D).
- The corners of a solid shape are called its vertices, the line segments/curves which form its skeleton are its edges and its flat surfaces are its faces.

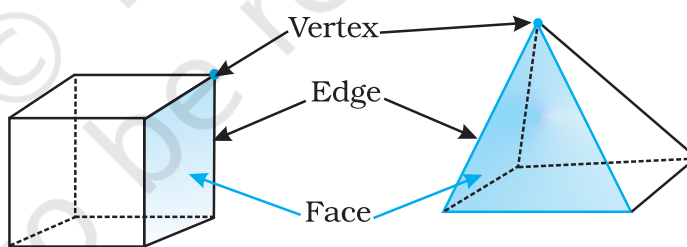


Fig. 12.1

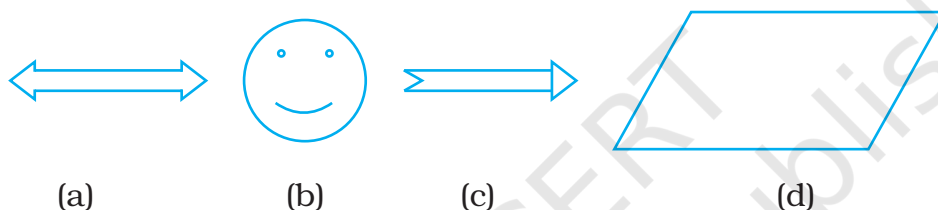
- A net is a skeleton-outline of a solid that can be folded to make the solid.
- Solid shapes can be drawn on a flat surface. This is called a 2-D representation of a 3-D solid (shape).
- Two types of sketches of a solid are possible:
 - (i) An oblique sketch which does not have proportional measurements.
 - (ii) An isometric sketch which is drawn on an isometric dot paper. In this sketch of the solid, the measurements are kept proportional.

- Different sections of a solid can be viewed in many ways:
 - By cutting or slicing, the shape, which would result in the cross-section of the solid.
 - By observing a 2-D shadow of a 3-D shape.
 - By looking at the shape from different positions-the front-view, the side-view and the top-view.

(B) Solved Examples

In Examples 1 to 3, there are four options, out of which one is correct. Choose the correct one.

Example 1: Which of the following is not a symmetrical figure?



Solution: Correct answer is (d).

Example 2: In the word “MATHS” which of the following pairs of letters shows rotational symmetry

- (a) M and T (b) H and S (c) A and S (d) T and S

Solution: Correct answer is (b).

Example 3: The angle of rotation for the figure 12.2 is

- (a) 45° (b) 60°
(c) 90° (d) 180°

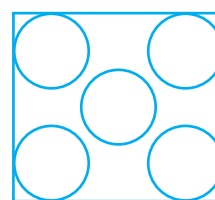


Fig. 12.2

Solution: Correct answer is (C)

In Examples 4 to 6, fill in the blanks to make it a true statement.

Example 4: The figure 12.3 has _____ vertices, _____ edges and _____ faces.

Solution: 10, 15, 7

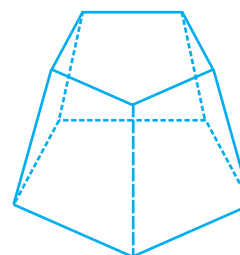


Fig. 12.3

Example 5: The adjoining net in Fig. 12.4 represents a _____.

Solution: Cube

Example 6: Rotation turns an object about a fixed point. This fixed point is called _____.

Solution: centre of rotation.

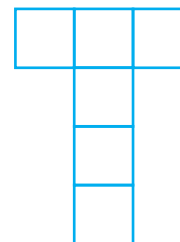


Fig. 12.4

In Examples 7 to 9, state whether the statements are True or False.

Example 7: A net of a 3-D shape is a sort of skeleton - outline in 2-D, which, when folded results in the 3-D shape.

Solution: True

Example 8: A regular pentagon has no lines of symmetry.

Solution: False

Translation	Rotation	Reflection
A translation slides a figure along the direction of a line without turning.	A rotation turns a figure around a point, called the centre of rotation .	A reflection flips a figure across a line to create a mirror image.

Example 9: Order of rotational symmetry for the figure 12.5 is 4.

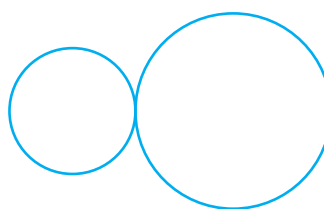


Fig. 12.5

Solution: False

- Solution:**
- (i) Rectangular Pyramid
 - (ii) Triangular Prism

Example 13: Construct a triangle PQR such that $PQ = 6$ cm, $QR = 7$ cm and $PR = 4.5$ cm.

Solution

- Steps:**
- (i) Draw a line segment PQ of length 6 cm.
 - (ii) With P as centre, draw an arc of radius 4.5 cm.
 - (iii) With Q as centre, draw an arc of radius 7 cm which intersects the previous arc at R.
 - (iv) Join PR and QR.

Then ΔPQR is the required triangle (Fig. 12.8).

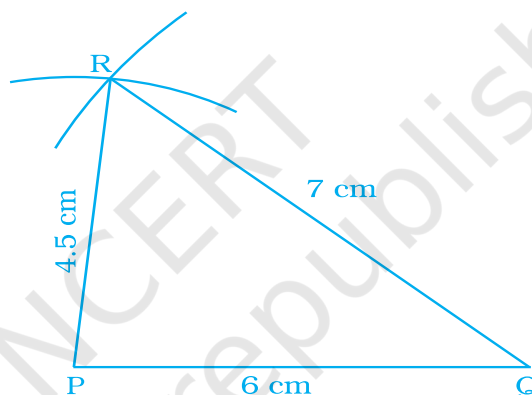


Fig. 12.8

Example 14: Draw the top, the front and the side views of the following solid figure made up of cubes.

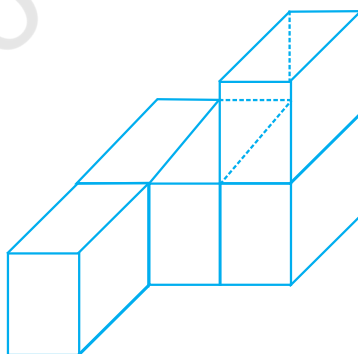


Fig. 12.9

Solution: Desired views are shown in Fig. 12.10 below

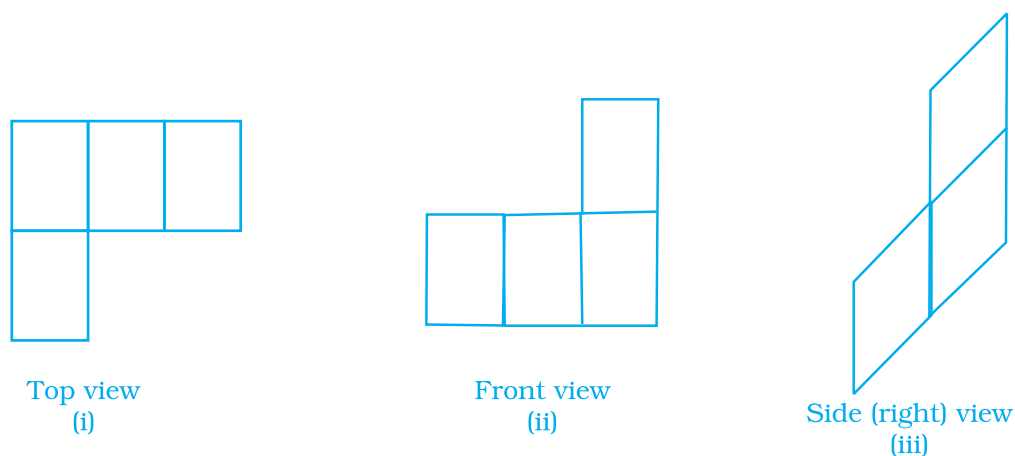
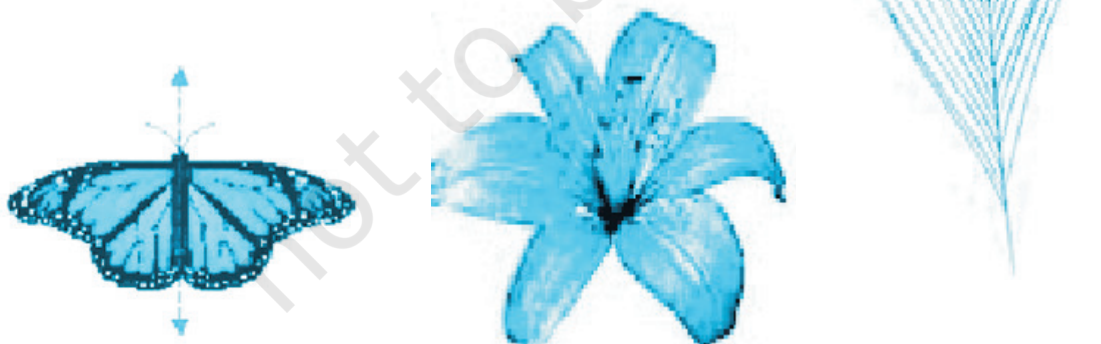


Fig. 12.10

Nature provides many beautiful examples of *symmetry*, such as the wings of a butterfly and a peacock or the petals of a flower. Symmetric objects have parts that are congruent.

A figure has **line symmetry** if you can draw a line through it so that the two sides are mirror images of each other. The line is called the **line of symmetry**.



Example 15: Given a line l and a point M on it draw a perpendicular MP to l where $MP = 5.2\text{cm}$ and a line q parallel to l through P .

Solution

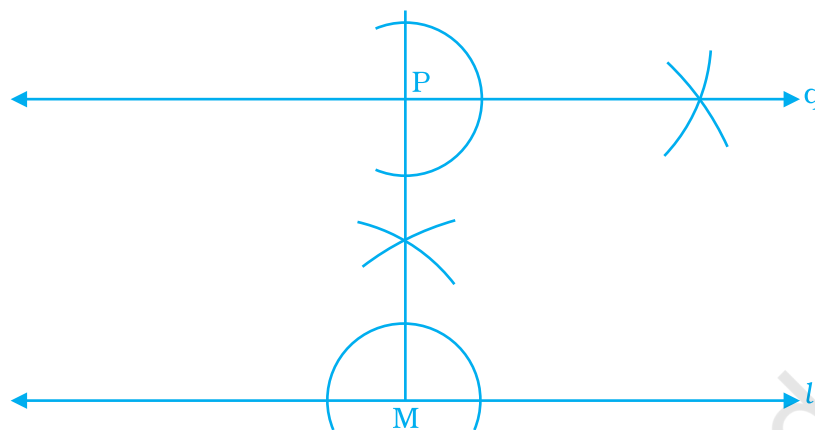


Fig. 12.11

- Steps :**
- (i) Draw a line l .
 - (ii) Take a point M on it.
 - (iii) Draw an angle of 90° at M with l which is perpendicular to l at M .
 - (iv) With M as centre and radius 5.2 cm, draw an arc which intersects the above perpendicular at point P . MP is the required perpendicular
 - (v) At P , draw an angle of 90° with PM and produce to make a line q .

Line q is the required line parallel to line l .

Application on Problem Solving Strategy



Example 16

Determine the number of edges, vertices and faces in the Fig. 12.12.

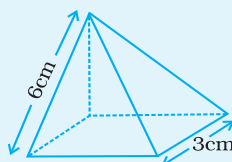


Fig. 12.12

Solution:



Understand and Explore the Problem

- What information is given in the question?
A square pyramid.
- What are you trying to find?
The number of edges, vertices and faces.
- Is there any information that is not needed?
The measure of the edges are not needed.



Plan a Strategy

- Recall the definitions of edges, vertices and faces of a 3-D figure and try to co-relate them to the figure given above.



Solve

- The different plain regions are called faces. Hence, there are 5 faces.
- The line segments formed, where the faces meet are called edges. Hence, there are 8 edges.
- Edges meet at a point which are called vertices. Hence, there are 5 vertices.
- Therefore, a square pyramid has 5 faces, 5 vertices and 8 edges.



Revise

- Try to find the number of vertices and edges of a cuboid. Can you see a pattern emerging based on your findings? you can observe that

$$F + V = E + 2$$

Where F,V,E denote number of faces, number of vertices and number of edges respectively of such solids. This is known as 'EULER'S FORMULA'. You'll study this concept in your next class.

Think and Discuss

Try to find the number of edges, vertices and faces in some more solids and explore the pattern, if any.

A figure has **rotational symmetry** if you can rotate the figure around some point so that it coincides with itself. The point is the centre of rotation, and the amount of rotation must be less than one full turn, or 360° .



7-fold and 6-fold rotational symmetry mean that the figures coincide with themselves 7 times and 6 times respectively, within one full turn.

7-fold rotational symmetry 6-fold rotational symmetry

(C) Exercise

In each of the Questions 1 to 26, there are four options, out of which one is correct. Choose the correct one.

1. A triangle can be constructed by taking its sides as:

(a) 1.8 cm, 2.6 cm, 4.4 cm	(b) 2 cm, 3 cm, 4 cm
(c) 2.4 cm, 2.4 cm, 6.4 cm	(d) 3.2 cm, 2.3 cm, 5.5 cm
2. A triangle can be constructed by taking two of its angles as:

(a) $110^\circ, 40^\circ$	(b) $70^\circ, 115^\circ$	(c) $135^\circ, 45^\circ$	(d) $90^\circ, 90^\circ$
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3. The number of lines of symmetry in the figure given below is:

(a) 4	(b) 8
(c) 6	(d) Infinitely many

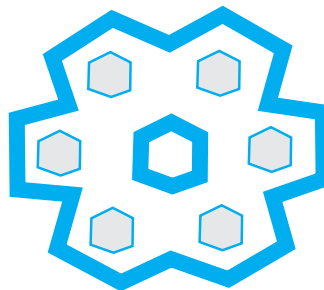


Fig. 12.13

Think and Discuss

1. **Explain** what it means for a figure to be symmetric.
2. **Tell** which capital letters of the alphabet have line symmetry.
3. **Tell** which capital letters of the alphabet have rotational symmetry.

4. The number of lines of symmetry in Fig. 12.14 is

- (a) 1 (b) 3
(c) 6 (d) Infinitely many



Fig. 12.14

5. The order of rotational symmetry in the Fig. 12.15 given below is

- (a) 4 (b) 8 (c) 6 (d) Infinitely many



Fig. 12.15

6. The order of rotational symmetry in the figure 12.16 given below is

- (a) 4 (b) 2
(c) 1 (d) Infinitely many

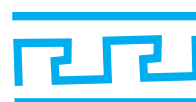


Fig. 12.16

7. The name of the given solid in Fig 12.17 is:

- (a) triangular pyramid (b) rectangular pyramid
(c) rectangular prism (d) triangular prism

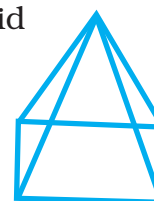


Fig. 12.17

Think and Discuss

1. **Give** a situation in which the front and side views of a figure would be the same.
2. **Explain** whether it is possible for all of the views of a figure to be congruent rectangles.

8. The name of the solid in Fig. 12.18 is:

- (a) triangular pyramid (b) rectangular prism
 (c) triangular prism (d) rectangular pyramid

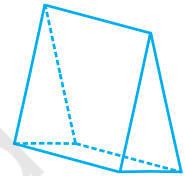


Fig. 12.18

9. All faces of a pyramid are always:

- (a) Triangular (b) Rectangular
 (c) Congruent (d) None of these

10. A solid that has only one vertex is

- (a) Pyramid (b) Cube (c) Cone (d) Cylinder

11. Out of the following which is a 3-D figure?

- (a) Square (b) Sphere (c) Triangle (d) Circle

12. Total number of edges a cylinder has

- (a) 0 (b) 1 (c) 2 (d) 3

13. A solid that has two opposite identical faces and other faces as parallelograms is a

- (a) prism (b) pyramid (c) cone (d) sphere

14. The solid with one circular face, one curved surface and one vertex is known as:

- (a) cone (b) sphere (c) cylinder (d) prism

15. If three cubes each of edge 4 cm are placed end to end, then the dimensions of resulting solid are:

- (a) $12\text{ cm} \times 4\text{ cm} \times 4\text{ cm}$ (b) $4\text{ cm} \times 8\text{ cm} \times 4\text{ cm}$
 (c) $4\text{ cm} \times 8\text{ cm} \times 12\text{ cm}$ (d) $4\text{ cm} \times 6\text{ cm} \times 8\text{ cm}$

16. When we cut a corner of a cube as shown in the figure 12.19, we get the cutout piece as :

- (a) square pyramid (b) trapezium prism
(c) triangular pyramid (d) a triangle



Fig. 12.19

17. If we rotate a right-angled triangle of height 5 cm and base 3 cm about its height a full turn, we get

- (a) cone of height 5 cm, base 3 cm
(b) triangle of height 5 cm, base 3 cm
(c) cone of height 5 cm, base 6 cm
(d) triangle of height 5 cm, base 6 cm

18. If we rotate a right-angled triangle of height 5 cm and base 3 cm about its base, we get:

- (a) cone of height 3 cm and base 3 cm
(b) cone of height 5 cm and base 5 cm
(c) cone of height 5 cm and base 3 cm
(d) cone of height 3 cm and base 5 cm

19. When a torch is pointed towards one of the vertical edges of a cube, you get a shadow of cube in the shape of

- (a) square (b) rectangle but not a square
(c) circle (d) triangle

20. Which of the following sets of triangles could be the lengths of the sides of a right-angled triangle:

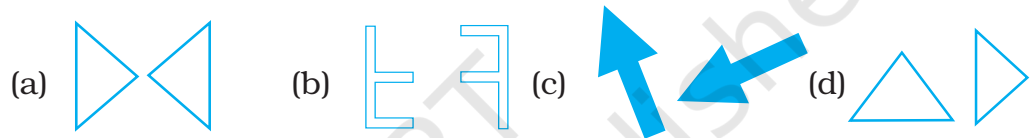
- (a) 3 cm, 4 cm, 6 cm (b) 9 cm, 16 cm, 26 cm
(c) 1.5 cm, 3.6 cm, 3.9 cm (d) 7 cm, 24 cm, 26 cm

- 21.** In which of the following cases, a unique triangle can be drawn
- (a) $AB = 4$ cm, $BC = 8$ cm and $CA = 2$ cm
 - (b) $BC = 5.2$ cm, $\angle B = 90^\circ$ and $\angle C = 110^\circ$
 - (c) $XY = 5$ cm, $\angle X = 45^\circ$ and $\angle Y = 60^\circ$
 - (d) An isosceles triangle with the length of each equal side 6.2 cm.

- 22.** Which of the following has a line of symmetry?



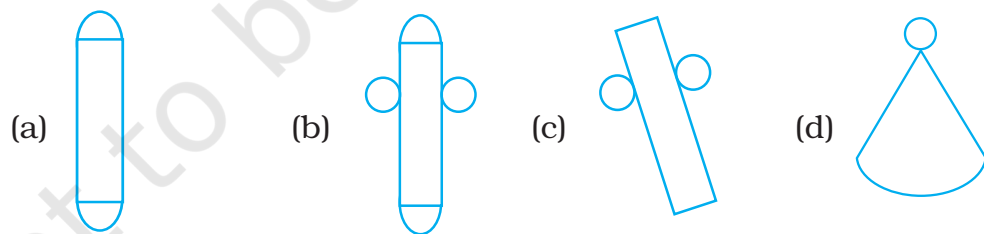
- 23.** Which of the following are reflections of each other?



- 24.** Which of these nets is a net of a cube?



- 25.** Which of the following nets is a net of a cylinder?

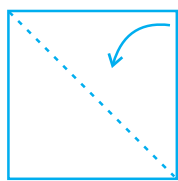


- 26.** Which of the following letters of English alphabets have more than 2 lines of symmetry?

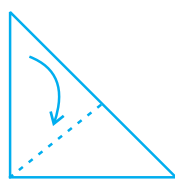


- 27.** Take a square piece of paper as shown in figure (1). Fold it along its diagonals as shown in figure (2). Again fold it as shown in figure (3). Imagine that you have cut off 3 pieces of the form of congruent

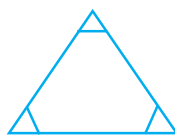
isosceles right-angled triangles out of it as shown in figure 4.



(1)



(2)

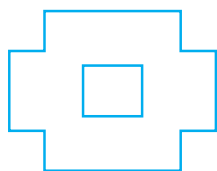


(3)

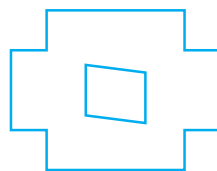


(4)

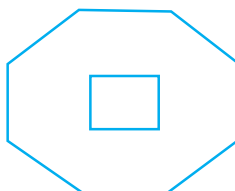
On opening the piece of paper which of the following shapes will you get?



(a)



(b)

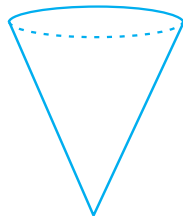


(c)

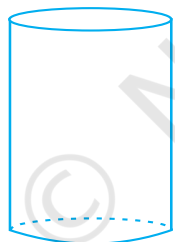


(d)

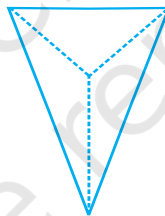
28. Which of the following 3-dimensional figures has the top, side and front as triangles?



(a)



(b)



(c)

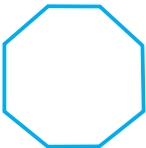


(d)

In Questions 29 to 58, fill in the blanks to make the statements true.

29. In an isosceles right triangle, the number of lines of symmetry is _____.
30. Rhombus is a figure that has _____ lines of symmetry and has a rotational symmetry of order _____.
31. _____ triangle is a figure that has a line of symmetry but lacks rotational symmetry.
32. _____ is a figure that has neither a line of symmetry nor a rotational symmetry.

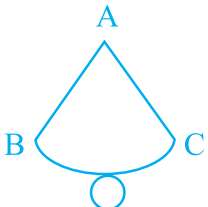
33. _____ and _____ are the capital letters of English alphabets that have one line of symmetry but they interchange to each other when rotated through 180° .
34. The common portion of two adjacent faces of a cuboid is called _____.
35. A plane surface of a solid enclosed by edges is called _____.
36. The corners of solid shapes are called its _____.
37. A solid with no vertex is _____.
38. A triangular prism has _____ faces, _____ edges and _____ vertices.
39. A triangular pyramid has _____ faces, _____ edges and _____ vertices.
40. A square pyramid has _____ faces, _____ edges and _____ vertices.
41. Out of _____ faces of a triangular prism, _____ are rectangles and _____ are triangles.
42. The base of a triangular pyramid is a _____.
43. Out of _____ faces of a square pyramid, _____ are triangles and _____ is/are squares.
44. Out of _____ faces of a rectangular pyramid _____ are triangles and base is _____.
45. Each of the letters H, N, S and Z has a rotational symmetry of order _____.
46. Order of rotational symmetry of a rectangle is _____.
47. Order of rotational symmetry of a circle is _____.
48. Each face of a cuboid is a _____.
49. Line of symmetry for an angle is its _____.
50. A parallelogram has _____ line of symmetry.

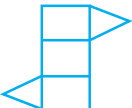
51. Order of rotational symmetry of  is _____.

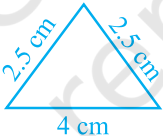
52. A _____ triangle has no lines of symmetry.

53. Cuboid is a rectangular _____.

54. A sphere has _____ vertex, _____ edge and _____ curved surface.

55.  is a net of a _____.
 → Circumference of circle = _____.

56.  is a net of a _____.

57. Order of rotational symmetry of  is _____.

58. Identical cubes are stacked in the corner of a room as shown below. The number of cubes that are not visible are _____.

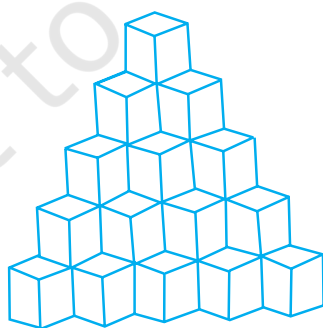


Fig. 12.20

In Questions from 59 to 92, state whether the statements are True or False.

59. We can draw exactly one triangle whose angles are 70° , 30° and 80° .
60. The distance between the two parallel lines is the same everywhere.
61. A circle has two lines of symmetry.
62. An angle has two lines of symmetry.
63. A regular hexagon has six lines of symmetry.
64. An isosceles trapezium has one line of symmetry.
65. A parallelogram has two lines of symmetry.
66. Order of rotational symmetry of a rhombus is four.
67. An equilateral triangle has six lines of symmetry.
68. Order of rotational symmetry of a semi circle is two.
69. In oblique sketch of the solid, the measurements are kept proportional.
70. An isometric sketch does not have proportional length.
71. A cylinder has no vertex.
72. All the faces, except the base of a square pyramid are triangular.
73. A pyramid has only one vertex.
74. A triangular prism has 5 faces, 9 edges and 6 vertices.
75. If the base of a pyramid is a square, it is called a square pyramid.
76. A rectangular pyramid has 5 rectangular faces.
77. Rectangular prism and cuboid refer to the same solid.
78. A tetrahedron has 3 triangular faces and 1 rectangular face.
79. While rectangle is a 2-D figure, cuboid is a 3-D figure.
80. While sphere is a 2-D figure, circle is a 3-D figure.
81. Two dimensional figures are also called plane figures.
82. A cone is a polyhedron.

83. A prism has four bases.
84. The number of lines of symmetry of a regular polygon is equal to the vertices of the polygon.
85. The order of rotational symmetry of a figure is 4 and the angle of rotation is 180° only.
86. After rotating a figure by 120° about its centre, the figure coincides with its original position. This will happen again if the figure is rotated at an angle of 240° .
87. Mirror reflection leads to symmetry always.
88. Rotation turns an object about a fixed point which is known as centre of rotation.
89. Isometric sheet divides the paper into small isosceles triangles made up of dots or lines.
90. The circle, the square, the rectangle and the triangle are examples of plane figures.
91. The solid shapes are of two-dimensional.
92. Triangle with length of sides as 5 cm, 6 cm and 11 cm can be constructed.
93. Draw the top, side and front views of the solids given below in Figures 12.21 and 12.22:

(i)



Fig. 12.21

(ii)

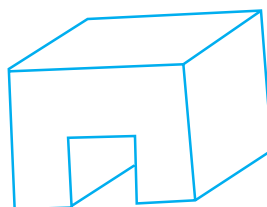
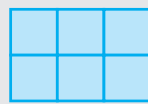
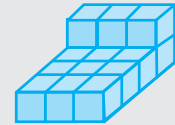


Fig. 12.22

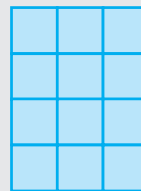
Three-dimensional figures often look different from different points of view. You can use centimetre cubes to help you visualize and sketch three-dimensional figures.

Activity

1. Use centimetre cubes to build the three-dimensional figure at right.
2. Now view the figure from the front and draw what you see. Then view the figure from the top and draw what you see. Finally, view the figure from the side and draw what you see.



Front



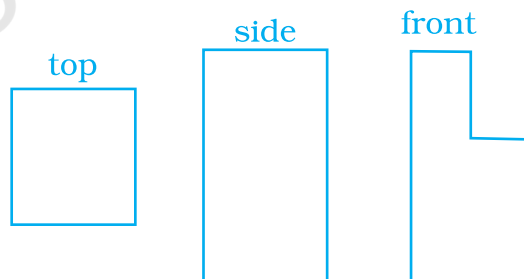
Top



Side

Think and Discuss

1. How many cubes did you use to build the three-dimensional figure?
 2. How could you add a cube to the figure without changing the top view?
 3. How could you remove a cube from the figure without changing the side view?
94. Draw a solid using the top, side and front views as shown below. [Use Isometric dot paper].



95. Construct a right-angled triangle whose hypotenuse measures 5 cm and one of the other sides measures 3.2 cm.
96. Construct a right-angled isosceles triangle with one side (other than hypotenuse) of length 4.5 cm.

97. Draw two parallel lines at a distance of 2.2 cm apart.
98. Draw an isosceles triangle with each of equal sides of length 3 cm and the angle between them as 45° .
99. Draw a triangle whose sides are of lengths 4 cm, 5 cm and 7 cm.
100. Construct an obtuse angled triangle which has a base of 5.5 cm and base angles of 30° and 120° .
101. Construct an equilateral triangle ABC of side 6 cm.
102. By what minimum angle does a regular hexagon rotate so as to coincide with its original position for the first time?
103. In each of the following figures, write the number of lines of symmetry and order of rotational symmetry.

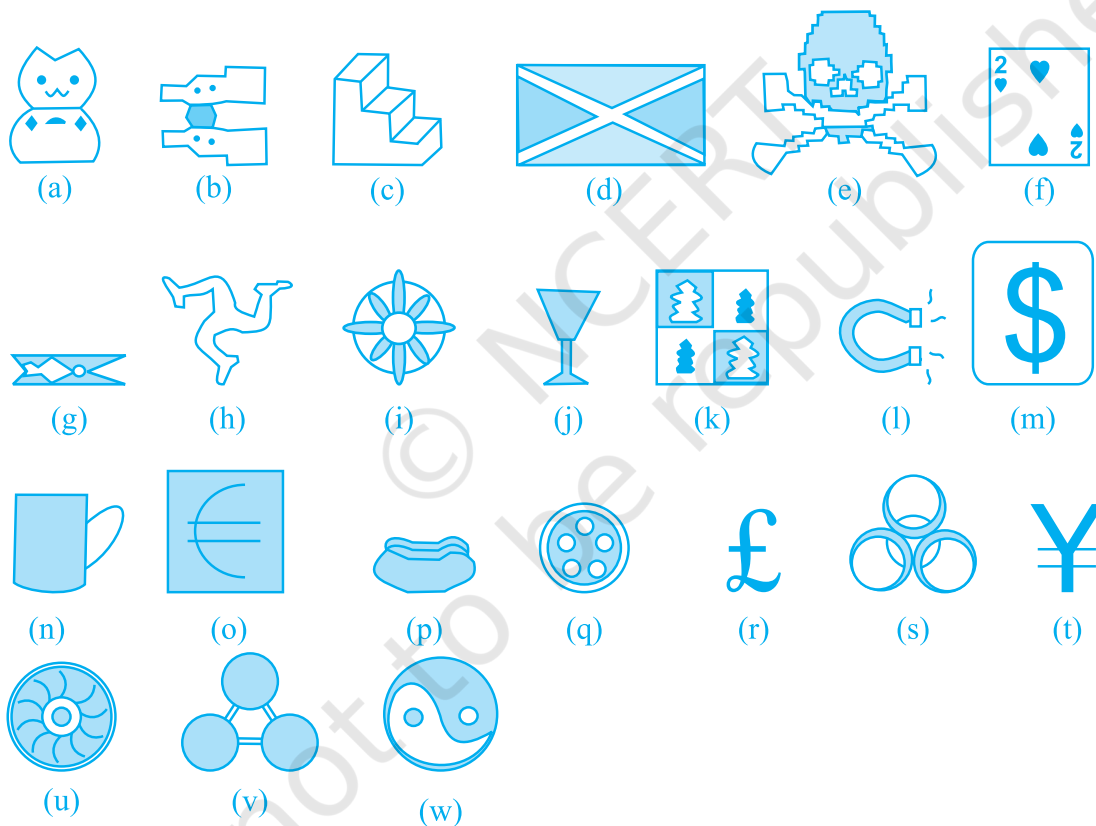


Fig. 12.23

[Hint: Consider these as 2-D figures not as 3-D objects.]

104. In the figure 12.24 of a cube,
 - (i) Which edge is the intersection of faces EFGH and EFBA?
 - (ii) Which faces intersect at edge FB?

- (iii) Which three faces form the vertex A?
- (iv) Which vertex is formed by the faces ABCD, ADHE and CDHG?
- (v) Give all the edges that are parallel to edge AB.
- (vi) Give the edges that are neither parallel nor perpendicular to edge BC.
- (vii) Give all the edges that are perpendicular to edge AB.
- (viii) Give four vertices that do not all lie in one plane.

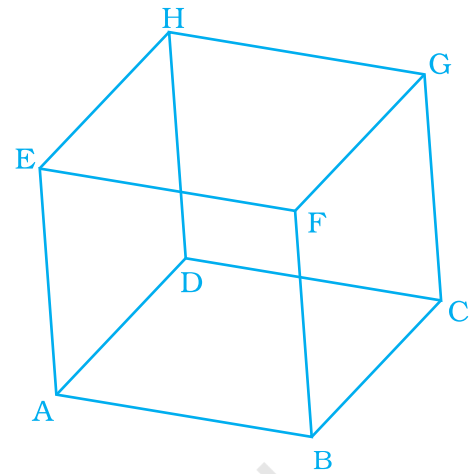


Fig. 12.24

- 105.** Draw a net of a cuboid having same breadth and height, but length double the breadth.
- 106.** Draw the nets of the following:
- (i) Triangular prism
 - (ii) Tetrahedron
 - (iii) Cuboid
- 107.** Draw a net of the solid given in the figure 12.25:

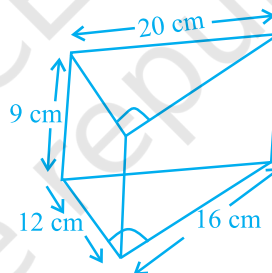


Fig. 12.25

- 108.** Draw an isometric view of a cuboid $6\text{ cm} \times 4\text{ cm} \times 2\text{ cm}$.
- 109.** The net given below in Fig. 12.26 can be used to make a cube.
- (i) Which edge meets AN?
 - (ii) Which edge meets DE?

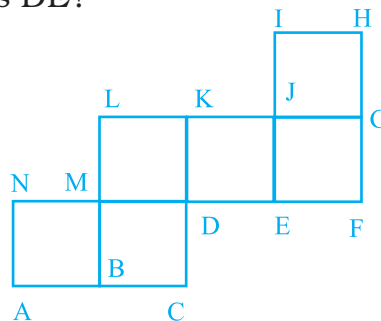


Fig. 12.26

- 110.** Draw the net of triangular pyramid with base as equilateral triangle of side 3 cm and slant edges 5 cm.
- 111.** Draw the net of a square pyramid with base as square of side 4 cm and slant edges 6 cm.
- 112.** Draw the net of rectangular pyramid with slant edge 6 cm and base as rectangle with length 4 cm and breadth 3 cm.

Activity

- 1.** Use centimetre cubes to build a figure that has the front, top and side views shown.



- 2.** You can build the figure by first making a simple figure that matches the front views.



- 3.** Now add cubes so that the figure matches the top view.



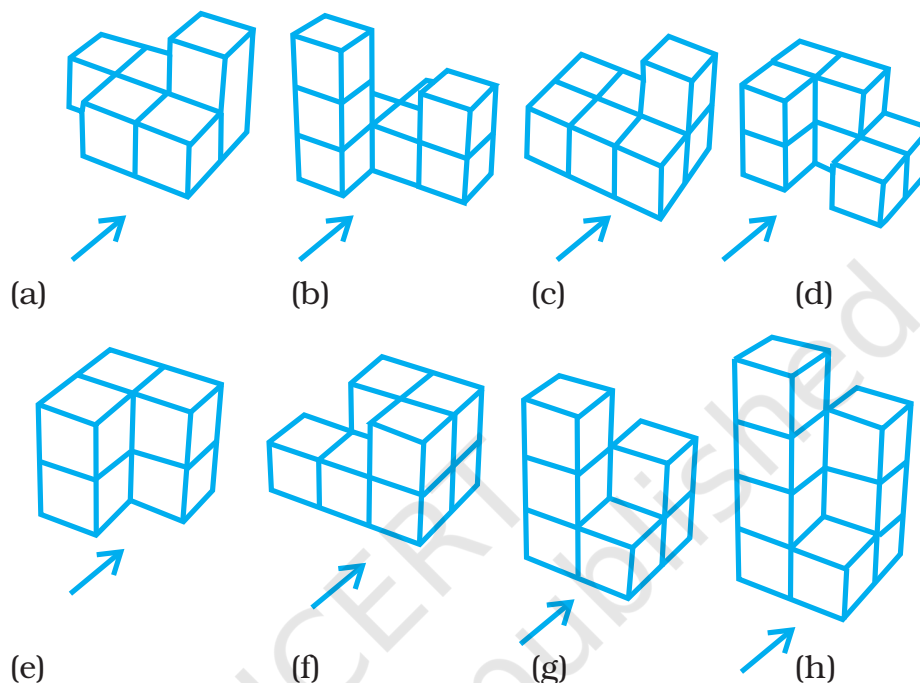
- 4.** Finally, remove cubes so that the figure matches the side view. Check that the front and top views are still correct for the figure that you built.



Think and Discuss

Discuss whether there is another step-by-step method for building the above figure. If so, is the final result the same.

- 113.** Find the number of cubes in each of the following figures and in each case give the top, front, left side and right side view (arrow indicating the front view).

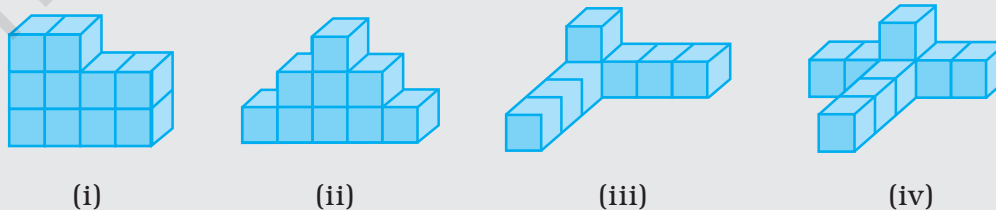


- 114.** Draw all lines of symmetry for each of the following figures as given below:



Try This

- 1.** Use centimetre cubes to build each three-dimensional figure given below. Then sketch the front, top and side views.



115. How many faces does Fig. 12.27 have?

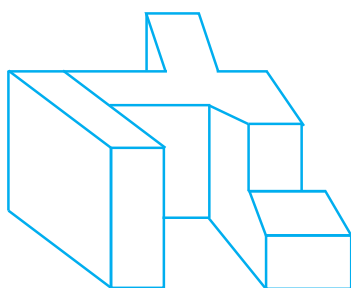
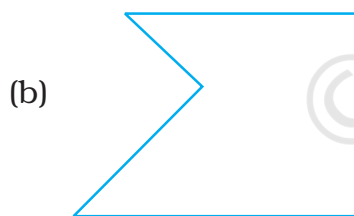
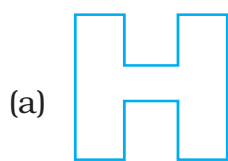
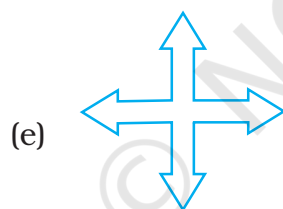
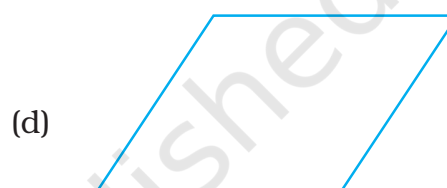
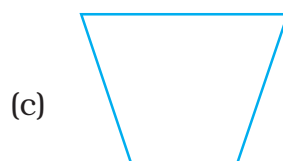
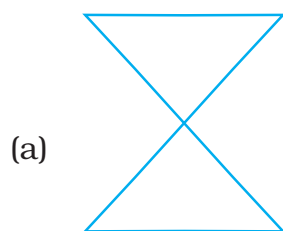


Fig. 12.27

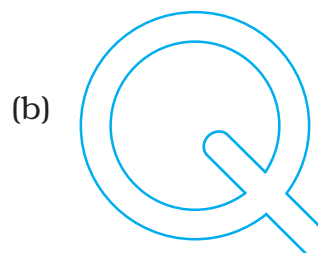
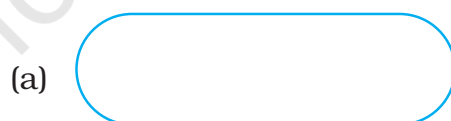
116. Trace each figure. Then draw all lines of symmetry, if it has.

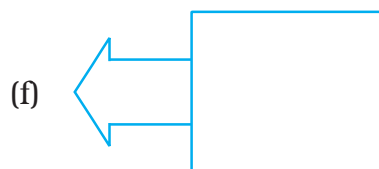
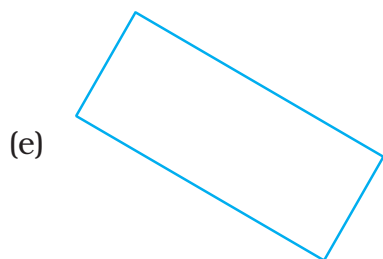
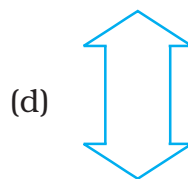
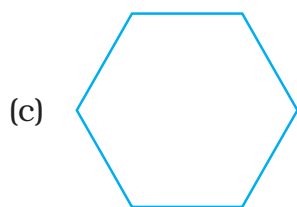


117. Tell whether each figure has rotational symmetry or not.

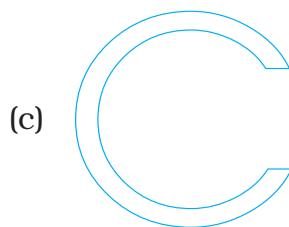
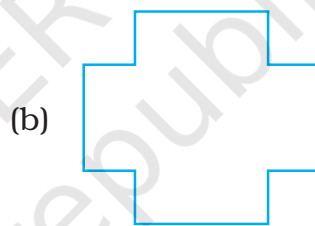
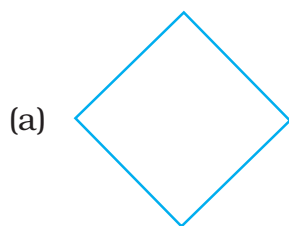


118. Draw all lines of symmetry for each of the following figures.





119. Tell whether each figure has rotational symmetry. Write yes or no.



120. Does the Fig. 12.28 have rotational symmetry?

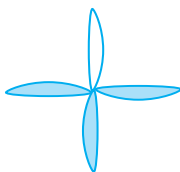


Fig. 12.28

- 121.** The flag of Japan is shown below. How many lines of symmetry does the flag have?

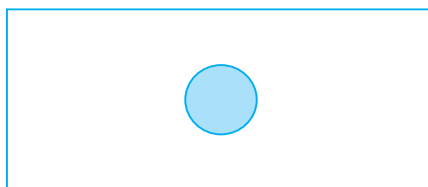
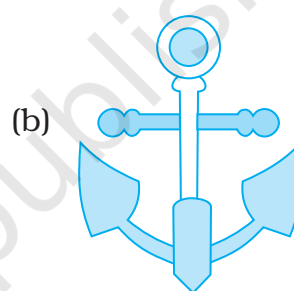
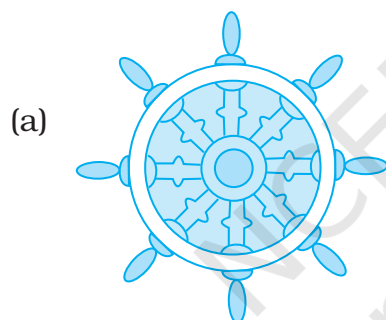
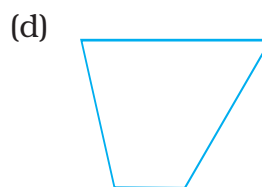
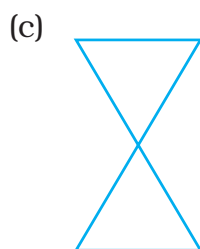
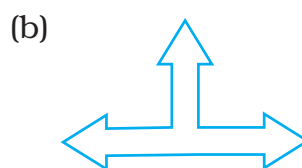
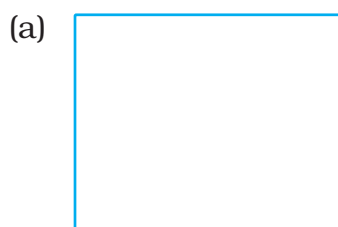


Fig. 12.29

- 122.** Which of the figures given below have both line and rotational symmetry?



123. Which of the following figures do not have line symmetry?



124. Which capital letters of English alphabet have no line of symmetry?

(D) Application

1. Crossword Puzzle

Solve the crossword and fill the given box across, downward as per the mentioned clue in the boxes.

Across

1. The sketch of a solid in which the measurements are kept proportional.
3. Two or more lines which remain apart at a constant distance, even if extended indefinitely.
5. The 3-D figure which has a Joker's cap.
7. A 2-D figure which has unlimited lines of symmetry and an infinite order of rotation.
9. The solid which has 5 faces- 3 of which are rectangles and 2 are triangles.

Down

2. The fixed point around which the object is rotated.
4. The solid shape which does not have a vertex or edge.
6. The line where two faces of a 3-D figure meet.
8. The skeleton 2-D figure which when folded results in a 3-D shape.
10. Shadow of a cube.

